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BOSTON UNIVERSITY

GRADUATE SCHOOL

Thesis

AN ANALYSIS OF THE MISTAKES MADE IN LONG DIVISION IN GRADES V TO VIII OF A CITY SCHOOL

Submitted by

James Louis Early

(B.S. of Ed., Boston University, 1925)

In partial fulfilment of requirements for

the degree of Master of Arts

1928

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FOREWORD

To Professor G. M. Wilson of Boston University School of Education, for inspiration and direction, acknowledgement is hereby gratefully made.

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INTRODUCTION

The writer is a firm believer in the statement that arithmetic to be serviceable must be characterized by an accuracy of 100%. That the work done in any of our classrooms is far below this standard must be admitted by all who are conversant with the actual schoolroom results. But that 100% accuracy even in the simplest fundamentals is the exception rather than the rule might seem to be a surprising statement. Yet anyone interested in this question has merely to read the results attained in city, state, or national surveys to realize that our school work in the fundamentals of arithmetic is far below the 100% level.

Is the 100% standard obtainable? For an individual, the answer is, of course, "yes". But for a class, a school, a whole town or city it is not so easy to say "yes". There is no question but with the proper kind of teaching, a class, a school, or a whole community can be brought nearer to the 100% ideal. The first thing necessary is to establish definitely 100% as the standard sought. There are many now who hold to the thesis that 100% accuracy in the fundamentals is necessary and can be achieved.

Granting that 100% accuracy is the ideal toward which we are to strive, we must then consider the ways and means that will enable us to reach our goal. We must admit that something better than our present procedure is necessary. A scientific study of the problem is called for. Simply giving more drill will not cure the trouble that confronts us. It must be drill of the right sort, systematic enough to provide opportunity

for the correction of the different kinds of mistakes which occur in the manipulating of numbers by children in the grades.

The foregoing paragraph has, as its logical sequence, the statement that we must know what are the kinds of mistakes that children make in operating upon numbers.

PURPOSE OF STUDY

The main purpose of this paper is to point out the nature and frequencies of the mistakes made in the most intricate and difficult of the fundamental processes; namely, long division. As the performing of long division is a complicated operation, the task at hand is not easy. Since the operation involves multiplication and subtraction as well as division, a check up on long division will be trebly valuable, for it affords a check up on multiplication and subtraction also.

The complications in the process of long division make it rather difficult to indicate in clear but fairly brief terminology the nature of the mistakes made. The writer in this study has striven to identify the errors made, and to indicate the types of errors in phraseology that will give the reader a clear-cut understanding of the nature of the mistakes made by the children.

PROCEDURE

For the purposes of this experiment two series of ten examples each were chosen. At the suggestion of Professor Wilson, a third series of ten examples containing types with easier divisors was added. Each series was given three times, with the exception of the third series, which, because it was added after the experiment was under way, was given only twice. The plan followed was to give one set of ten each week until all sets were given, and then repeat the procedure until all sets were given three times (excepting the third set). Thus, the experiment lasted over a period of eight weeks, during which the teachers could, if they so desired, give the children any needed drill on long division. The identical examples used in the experiment were not to be used for drill purposes but ones of similar type were allowable. It will be noted that successive trials of the same set were at intervals of three weeks so that there was no danger of pupils remembering a particular example. In fact, the results of the study seem to show that the sets were new each time the children did them. By using the same set more than once, it made it possible to trace a pupil's variations, to note improvement or deterioration in an identical task. In Table I on a succeeding page the variations of the pupils will be found.

The thirty examples selected were chosen with great care. No decimals were inserted, in order to avoid complications caused by the decimal point. To further confine the range of errors, the pupils were instructed to put as their answer,

first, the quotient; second, the word "and" followed by the remainder with the letter "r" after it; i.e., 27 ÷ 6 = 4 and 3r. If due to his previous training the pupil did express his remainder as the numerator of a fraction with the divisor as the denominator, the example was called correct whether reduced or not. It was the intent of the writer to avoid in this particular study any mistakes involved in the reduction of fractions. That is to say, in the whole study the emphasis was placed on identifying and tabulating the mistakes made in straight division of whole numbers.

EXAMPLES USED

Below are given the thirty examples used in this experiment. Although, when given to the children, they were broken into sets of ten, they are numbered here for convenience from one to thirty. The examples, as the reader will notice, provide for the testing of the difficulties inherent in long division, and for the use of as many of the multiplication and subtraction facts as possible.

1.	13)165	12 and 9r	Answer
2.	17)109	6 and 7r	"
3.	19)385	20 and 5r	"
4.	21)248	11 and 17r	"
5.	16)583	36 and 7r	"
6.	14)822	58 and 10r	"
7.	37)610	16 and 18r	"
8.	59)327	5 and 32r	"
9.	18)7,502	416 and 14r	"
10.	63)5,261	83 and 32r	"
11.	31)3,224	104 and 0r	"
12.	58)1,700	29 and 18r	"
13.	49)4,920	100 and 20r	"
14.	69)7,528	109 and 7r	"
15.	73)15,344	210 and 14r	"
16.	98)53,872	549 and 70r	"
17.	324)5,876	18 and 44r	"
18.	287)75,002	261 and 95r	"
19.	306)612,669	2,002 and 57r	"
20.	82)2,357,648	28,571 and 66r	"
21.	11)3,864	351 and 3r	"
22.	14)7,948	567 and 10r	"
23.	12)4,859	404 and 11r	"
24.	11)2,295	208 and 7r	"
25.	18)7,562	420 and 2r	"
26.	15)9,867	657 and 12r	"
27.	11)10,382	943 and 9r	"
28.	65)51,322	789 and 37r	"
29.	26)2,599	99 and 25r	"
30.	11)8,370	760 and 10r	"

TYPES OF EXAMPLES

The examples were selected so as to give ample opportunity for testing difficulties that occur in the most troublesome types, such as:

a. Finding the first quotient figure when the number of times that the first divisor figure is contained in the first dividend figure is not the true quotient figure. Examples 5, 6, 7, 9, 18, 22, 23, 25, 26, and 30 test this particular difficulty.

b. Finding first quotient figure when more figures in the dividend must be used than the number of figures in the divisor. Examples 2, 8, 10, 12, 15, 16, 20, 27, 28, and 29 test this difficulty.

c. The quotient which has a zero as one of its figures between other figures. Examples 11, 14, 19, 23, and 24 test this difficulty.

d. The quotient which has a zero at the end. Examples 3, 13, 15, 25, and 30 test this difficulty.

In Table IV will be found data showing relative difficulty of the thirty examples as shown by the frequency of incorrect examples.

VARIATIONS OF PUPILS DOING SUCCESSIVE SETS

Before going on to the tabulation of the particular errors made and their frequencies (see Table V), it will be interesting to note the variations of the pupils as they did successive sets. The variation is shown in Table I which immediately follows.

The table is arranged by grades to show the positive and negative variation which occurred as pupils did a set a second or third time. The columns marked 2d show the variation between the first and second trials. The columns marked 3d show the variations between the second and third trials. All variations are based on number of correct examples. For instance, a variation of +1 means one more correct example; 0 means same score as before; and -1 means one less correct example. The variations are listed at left of table and the numbers under each grade numeral V, VI, VII, and VIII indicate the number of pupils in that grade whose variation was the one listed at the left of the table.

Table Ia (first ten examples)

Variations of Pupils in Doing Successive Sets

(Variations--"Var." at left; Roman numerals refer to grades; numbers under numerals indicate number of pupils of that grade whose variation is indicated at left of table)

	V	V	VI	VI	VII	VII	VIII	VIII	Total	
Var.	2d	3d	2d*	3d	2d"	3d	2d#	3d	2d	3d
+6	2	0	0	0	1	0	0	0	3	0
+5	1	1	1	0	0	1	0	0	2	2
+4	5	0	2	0	0	1	1	1	8	2
+3	5	5	2	1	2	4	4	0	13	10
+2	6	6	4	1	5	8	3	2	18	17
+1	11	8	8	3	10	10	14	16	43	37
0	28	23	22	35	14	21	28	28	92	107
-1	13	15	5	8	15	7	8	10	41	40
-2	0	4	7	1	2	1	0	2	9	8
-3	2	0	0	3	1	1	0	0	3	4
-4	0	1	0	0	0	0	0	1	0	2
-5	0	5	0	0	0	0	1	0	1	5
-6	0	2	0	0	0	0	0	0	0	2

*One pupil in grade VI had a variation of +10; as the first time he did the set, he put his divisor as his quotient in the answer, every time.

"One pupil in grade VII had a variation of +8. He was coached in long division after his first trial, as was the pupil who had a variation of +6.

#One pupil in grade VIII had a variation of +10.

Table Ib (second ten examples)

Variations in Doing Successive Sets

	V	V	VI	VI	VII	VII	VIII	VIII	Total	
Var.	2d	3d	2d	3d	2d	3d	2d	3d	2d	3d
+6	0	0	0	0	1	1	0	0	1	1
+5	0	3	0	0	0	0	0	0	0	3
+4	2	2	3	0	1	1	1	1	7	4
+3	3	5	3	1	0	0	1	4	7	10
+2	8	9	6	4	11	5	6	9	31	27
+1	10	14	7	14	16	14	10	14	43	56
0	14	12	27	18	7	21	20	16	68	67
-1	10	8	6	12	8	6	12	10	36	36
-2	8	4	4	5	4	5	4	4	20	18
-3	4	3	0	0	1	1	2	1	7	5
-4	1	1	0	0	1	0	0	0	2	1
-5	1	0	0	0	1	0	0	0	2	0
-6	0	0	0	0	0	0	1	0	1	0

Table Ic (third ten examples, done only twice)

	V	VI	VII	VIII	Total
Var.	2d	2d	2d	2d	2d
+6	1	1	0	0	2
+5	0	0	0	0	0
+4	2	1	1	0	4
+3	2	5	1	1	9
+2	10	3	4	7	24
+1	10	10	15	5	40
0	22	28	28	38	116
-1	7	9	3	15	34
-2	1	0	1	1	3
-3	2	0	0	0	2
-4	1	0	1	0	2

Comments on The Variation Tables

Since the pupils who did the thirty examples in long division were not exactly the same ones every time a set was given, the total number of pupils for any certain grade is not the same any two times.

Those scored as 0 in variation include pupils who had perfect scores successive times, so the number having 0 variation is, of course, comparatively large.

SUMMARIES OF VARIATIONS

From the Variation Tables can be found the net gain or loss which occurred as the successive sets were done. It is not the intention of the writer to make all the comparisons which might be made from this set of statistics as the main object of this paper is to study the kinds and frequencies of the errors made by the children. However, it will be interesting to note whether or not the whole school showed any or much improvement as the children went on with the sets. The total positive variation compared with the total negative variation will give the answer to this question:

In the first ten examples, second trial:

Total + variation	206
Total - variation	73
Net + variation	133 for 236 pupils

In the first ten examples, third trial:

Total + variation	119
Total - variation	113
Net + variation	6 for 236 pupils

In the second ten examples, second trial:

Total + variation	160
Total - variation	121
Net + variation	39 for 225 pupils

In the second ten examples, third trial:

Total + variation	177
Total - variation	91
Net + variation	86 for 228 pupils

In the third ten examples, second trial:

Total + variation	143
Total - variation	54
Net + variation	89 for 236 pupils

From the summaries above, it will be noted that on every successive trial there was a net gain for the school as a whole but it is somewhat disheartening to realize that there were a great many pupils who not only did not hold their own on successive trials but who actually deteriorated and in some cases to a considerable extent. This fact brings us face to face with a great difficulty to be surmounted if we are to achieve 100% accuracy for a class or school--namely, we must not only bring those of low grade up to the desired level but we must also keep those who have reached a certain level from dropping below that level.

The great number of pupils who showed variation one way or the other would seem to show that we cannot depend upon children as now generally taught to exhibit the same degree of accuracy on successive trials.

The writer has kept a record of the results of each pupil who participated in this contest, with his or her variations. As most of these pupils, other than those of grade VIII will be in the same building next year, it will be possible to prepare, for the teachers, lists of pupils needing especial attention in long division; and for the writer to follow up the record of individual pupils to see if improvement is achieved next year. It is the hope of the writer, also, that from the results of this study, the teachers can be given some suggestions that will aid them in going about their work more scientifically.

OCCURRENCE OF INCORRECT EXAMPLES

Table II

Table II shows the number of times each example was done incorrectly in each of two trials by pupils of grades VII and VIII. The "1st" and "2d" heading different columns refer to first and second trials of the same examples. There were a and b sections of each grade.

(Occurrence of incorrect examples, each two trials)

	VIIa			VIIb			VIIIa			VIIIb			Total		
Ex.	1st	2d	Total	1st	2d	Total	1st	2d	Total	1st	2d	Total	1st	2d	Total
1	0	3	3	4	0	4	1	1	2	1	0	1	6	4	10
2	2	1	3	5	4	9	1	0	1	4	2	6	12	7	19
3	1	1	2	6	6	12	0	0	0	7	1	8	14	8	22
4	5	0	5	7	5	12	1	0	1	2	4	6	15	9	24
5	1	2	3	4	4	8	1	0	1	4	1	5	10	7	17
6	2	0	2	4	6	10	2	2	4	5	3	8	13	11	24
7	3	2	5	2	5	7	0	1	1	4	5	9	9	13	22
8	2	1	3	8	3	11	1	1	2	3	5	8	14	10	24
9	2	3	5	7	10	17	5	0	5	4	4	8	18	17	35
10	4	4	8	3	8	11	7	5	12	9	7	16	23	24	47
11	2	0	2	2	0	2	1	0	1	5	0	5	10	0	10
12	5	3	8	3	7	10	3	3	6	8	4	12	19	17	36
13	0	0	0	6	4	10	1	1	2	6	4	10	13	9	22
14	2	3	5	4	7	11	2	2	4	4	4	8	12	16	28
15	3	2	5	6	10	16	1	2	3	3	4	7	13	18	31
16	11	10	21	13	13	26	5	7	12	6	7	13	35	37	72
17	7	2	9	9	9	18	2	2	4	7	3	10	25	16	41
18	6	9	15	16	16	32	9	8	17	15	7	22	46	40	86
19	6	0	6	12	8	20	3	7	10	4	2	6	25	17	42
20	10	7	17	18	10	28	8	12	20	10	8	18	46	37	83
21	1	2	3	1	0	1	2	2	4	1	2	3	5	6	11
22	1	0	1	2	1	3	0	3	3	2	2	4	5	6	11
23	1	0	1	1	1	2	1	0	1	2	1	3	5	2	7
24	2	0	2	2	1	3	0	1	1	1	2	3	5	4	9
25	4	1	5	7	2	9	3	0	3	5	1	6	19	4	23
26	2	0	2	2	1	3	2	1	3	2	3	5	8	5	13
27	1	0	1	1	2	3	1	0	1	2	2	4	5	4	9
28	7	3	10	8	6	14	4	6	10	4	3	7	23	18	41
29	3	2	5	0	3	3	2	2	4	4	3	7	9	10	19
30	4	2	6	4	3	7	1	2	3	6	2	8	15	9	24
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EXAMPLES REARRANGED ACCORDING TO DIFFICULTY

The thirty examples arranged in order of difficulty as shown in Table II. The number to the right of the example is the number of times it was done incorrectly, counting all the pupils in the first two trials.

Revised Order	Examples		Times Wrong	Original Order
1	12)4,859	-	7 wrong	(Ex 23)
2	11)2,295	-	9 "	(" 24)
3	11)10,382	-	9 "	(" 27)
4	13)165	-	10 "	(" 1)
5	31)3,224	-	10 "	(" 11)
6*	11)3,864	-	11 "	(" 21)
7	14)7,948	-	11 "	(" 22)
8	15)9,867	-	13 "	(" 26)
9	16)583	-	17 "	(" 5)
10	17)109	-	19 "	(" 2)
11	26)2,599	-	19 "	(" 29)
12	19)385	-	22 "	(" 3)
13	37)610	-	22 "	(" 7)
14	49)4,920	-	22 "	(" 13)
15	18)7,562	-	23 "	(" 25)
16	21)248	-	24 "	(" 4)
17*	14)822	-	24 "	(" 6)
18	59)327	-	24 "	(" 8)
19*	11)8,370	-	24 "	(" 30)
20	69)7,528	-	28 "	(" 14)
21	73)15,344	-	31 "	(" 15)
22	18)7,502	-	35 "	(" 9)
23	58)1,700	-	36 "	(" 12)
24	324)5,876	-	41 "	(" 17)
25	65)51,322	-	41 "	(" 28)
26	306)612,669	-	42 "	(" 19)
27*	63)5,261	-	47 "	(" 10)
28	98)53,872	-	72 "	(" 16)
29	82)2,357,648	-	83 "	(" 20)
30	287)75,002	-	86 "	(" 18)

*Examples starred are discussed on the next page.

COMMENTS ON APPARENT DIFFICULTY OF EXAMPLES

The preceding list, page 15, affords some surprises. It is difficult to understand why example 6 in the arranged list, which was example 21 in the set as given, should prove more difficult than example 3 in the arranged list, which was example 27 as given--why 3,864 ÷ 11 (351 and 3r Ans.) should be missed more times than 10,382 ÷ 11 (943 and 9r Ans.) is difficult to understand.

Again example 17 in the arranged list, which was given as example 6, was missed over twice as many times as example 7 in the arranged list, which was example 22 when given. Example 17 is 822 ÷ 14 (58 and 10r Ans.); example 7 is 7,948 ÷ 14 (567 and 10r Ans.). One would naturally expect that example 7 would be missed more times than example 17.

Examples 19 and 27 in the arranged list owe their comparatively low positions partly to the fact that each was at the end of a given set. Example 19 was the tenth in a given set of ten and example 27 was in the same position. The fact that each was at the end of its respective set had something to do with the number of times it was done wrong. Example 29 in the arranged set was also given as the last one in a set of ten, but probably the fact that its dividend is so large caused it to be done wrong so many times.

The writer intends to have these examples given in succeeding years and from the data then gathered, rearrange the examples according to difficulty.

For this contest, as about 260 pupils participated each time, although not always the same 260, each example was done, in two trials, over 500 times. While that is not enough to furnish materials for standardization, it was sufficient to give the writer an idea of how one particular school succeeded with certain examples.

The next table, Table III, shows the kind and frequencies of errors made. Comment on the data shown in this table will be found on succeeding pages.

Table III

Kinds and Frequencies of Errors Made	First Ten VIII	VII	VI	V	Total	Second Ten VIII	VII	VI	V	Total	Third Ten VIII	VII	VI	V	Total	TOTAL
Copying wrong figures	12	17	14	24	67	9	9	7	35	60	7	5	3	4	19	146
Mistaking own figures	6	12	7	2	27	8	23	7	6	44	5	6	4	4	19	90
Division:																
Dividend, annexing 0 to	5	8	0	0	13	3	12	0	0	15	1	2	0	0	3	31
Dividend, figure brought down twice	0	0	0	1	1	1	0	0	1	2	1	0	0	0	1	4
Dividend, not bringing down next figure	2	3	7	5	17	18	25	7	35	85	1	2	3	7	13	115
Dividend, wrong figure brought down	2	3	3	2	10	3	5	7	6	21	1	5	3	2	11	42
Dividend, using more places than needed	1	0	2	2	5	0	2	0	3	5	1	0	0	0	1	11
Divisor put as remainder in answer	1	2	0	1	4	1	1	0	0	2	0	0	0	0	0	6
Divisor put as quotient in answer	1	3	9	1	14	3	2	0	0	5	1	1	0	0	2	21
Quotient, extra figure annexed	0	0	0	1	1	0	0	0	1	1	0	0	0	0	0	2
Quotient, extra figure inserted	0	0	2	1	3	2	4	5	0	11	0	0	0	0	0	14
Quotient, figure left out	0	0	0	0	0	5	6	2	0	13	1	3	1	0	5	18
Quotient, last figure where 0 not set down	3	3	15	5	26	9	12	2	5	28	7	5	0	3	15	69
Quotient, last figure other than 0 not set down	1	1	2	2	6	0	0	4	2	6	1	0	1	1	3	15
Quotient, wrong figure set down but right one used	3	6	7	3	19	4	5	10	1	20	1	2	3	2	8	47
Quotient figure too small	15	14	15	51	95	16	26	18	45	105	3	6	10	16	35	235
Jumbling process	8	0	1	3	12	7	5	3	10	25	1	3	3	3	10	47
Multiplication-facts no carrying	12	12	16	19	59	12	13	10	10	45	4	5	7	6	24	128
Multiplication-facts with carrying	13	9	23	18	63	31	52	31	54	168	17	17	13	22	69	300
Subtraction-adding one fig. instead	0	4	0	0	4	0	1	1	1	3	0	0	0	0	0	7
Facts-up to ten	12	5	4	5	26	11	11	6	16	44	8	1	5	7	21	91
Facts-ten or over	7	10	9	17	43	12	24	24	35	95	7	5	3	8	23	161
Decreasing next figure although no borrowing	3	12	3	5	23	25	20	16	14	75	1	2	3	1	7	105
Borrowing although subtrahend is larger than minuend	0	2	0	10	12	3	4	4	14	25	0	0	1	1	2	39
Left-hand figures not subtracted	1	10	7	12	30	11	21	13	9	54	2	2	3	1	8	92
Not decreasing next figure for one borrowed	0	3	7	8	18	5	13	16	17	51	1	0	2	5	8	77
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COMMENTS ON ERRORS MADE

Comments on Table V

It will be noted that the data are arranged to show the frequency of each kind of mistake, by grades, for each of the three sets of ten examples each. As the results of all the trials, three for each of the first two sets of ten and two for the third set, were used in the making of this table, the frequencies under the third ten are not in true proportion to those of the other two sets. But the purpose of this particular summary is first to identify specific kinds of mistakes made, and next to list the total number of times each specific mistake was made. The table served this purpose and for this study, at least, the comparison between the frequencies in the different sets is not important. However, it will be seen that the second ten examples, which contained harder computations, furnished the majority of the mistakes under most of the types of errors.

In all, 1913 mistakes were tabulated. Of these, 47, or about 2½% were of such a nature as to defy definite identification unless the number of types of errors was made unwieldy. Errors listed under "Jumbling process" were of this nature. A few illustrations will serve to show the kinds of errors that were listed under "Jumbling process".

a.

```
       xx
      181
  69)7528
     69
      628
      552
       76
       69
        7
```

In this example, the pupil brought down the 2, and then, instead of putting 0 as the next quotient figure, he brought down the 8. Next, selecting 8 as the next quotient figure, he was left with 76, a remainder greater than his divisor. Hence, he puts 1 as the next ~~divisor~~ QUOTIENT figure. The correct answer, 109 and 7r, bears some relation to 181 and 7r, but was obtained by a jumbling of the process.

b.

```
        x
      120
  49)4920
     49
      020
```

Putting the 2 in the quotient of example b. is another illustration of "jumbling". (Correct answer, 100 and 20r).

A third illustration of "jumbling" is found in example c. below.

c.

```
         x
       212
  73)15344
     146
       74
       73
       14
       14x
```

In this example the pupil seemingly put any figure that came to his mind, instead of 0 which was correct, as the last quotient figure. Then, to round things off nicely, he put 14 as the last partial product. (Correct answer, 210 and 14r).

The phrasing of another type of error which might seem to be a little too general, "Quotient figure too small", was used to cover only such cases as when the pupil, starting off with a quotient figure smaller than the correct one, went serenely on his way in spite of the fact that his remainder was larger than his divisor. In cases where this particular mistake was made on other than the last quotient figure, it was accompanied at times by the mistake of failing to subtract the left-hand figures. In such cases a mistake under the heading, "Left-hand figures not subtracted", was scored instead of one under "Quotient figure too small".

No heading such as "Quotient figure too large" was used because in every case in which a pupil had a quotient figure too large he made one of two types of errors to offset his original poor choice. He either made a mistake in multiplying, which obtained for him a partial product smaller than the used part of the dividend; or he multiplied correctly, getting a partial product larger than the used part of the dividend, but subtracted by borrowing even though his subtrahend was larger than his minuend--a sort of "illegal" borrowing. Therefore, mistakes originating from starting off with a quotient figure too large are listed under headings showing mistakes in multiplying or in subtraction.

The phrasing of the other types of errors is sufficiently clear as to require no comment.

SUMMARY OF ERRORS MADE

The errors made may be divided into four groups:

1. Carelessness in copying figures
2. Mistakes other than in multiplication or subtraction
3. Mistakes in multiplication
4. Mistakes in subtraction

Of each of these four general groups the frequency of errors was as follows:

1. Carelessness	236
2. Division	677
3. Multiplication	428
4. Subtraction	572
Total	1913

Summarizing a little differently, the mistakes made in division process itself totaled 677, and other mistakes totaled 1236, or almost twice as many mistakes made because of inaccuracies not due to difficulties in the process of long division as mistakes due to difficulties in the process itself.

The general groups of errors are subdivided in the table into more specific errors. It will be interesting to note, in order, the errors that occurred with the greatest frequency. A summary listing the more common mistakes with the approximate per cent of their occurrence follows.

SUMMARY OF ERRORS WITH PER CENTS OF TYPES

		%
Multiplication facts, with carrying	300	15.7
Quotient figure too small	235	12.2
Subtraction facts, ten or over	161	8.4
Copying wrong figures	146	7.6
Multiplication facts, no carrying	128	6.7
Dividend, not bringing down next figure	115	6.0
Decreasing next figure, although no borrowing	105	5.5
Left-hand figures not subtracted	92	4.8
Subtraction facts, up to ten	91	4.8
Mistaking own figures	90	4.7
Not decreasing next figure for one borrowed	77	4.0
Quotient, last figure when 0, not set down	69	3.6
Quotient, wrong figure set down, but right one used	47	2.5
Jumbling process	47	2.5
Dividend, wrong figure brought down	42	2.2
Borrowing, although subtrahend is larger than minuend	39	2.0
Dividend, annexing 0 to	31	1.6
Divisor put as quotient in answer	21	1.1
Quotient figure left out	18	1.0
Quotient, last figure other than 0, not set down	15	0.8
Quotient, extra figure inserted	14	0.7
Dividend, using more places than needed	11	0.6
Subtraction, adding one figure instead	7	0.4
Divisor put as remainder in answer	6	0.3
Dividend figure brought down twice	4	0.2
Quotient, extra figure annexed	2	0.1
	1913	100.0%

HELPFUL SUGGESTIONS GAINED FROM STUDY OF ERRORS MADE

A summary of this nature permits one to put a finger on the biggest troubles encountered by the pupils in doing long division. The summary indicates that drill work of several types is necessary. The writer would begin by putting most of the drilling on the mistake which occurred the greatest number of times, which would mean a systematic drill on the multiplication facts with carrying, such as (6 x 8) + 5 = ?, etc. And so on with the other specific errors. Drill of a few minutes should be had every day.

Insistence on the proving of the examples, would, without question, promote accuracy. Yet, in the case of a pupil starting off with a quotient figure too small, proving by multiplication did not help in several instances; for a pupil making such an error simply added on his remainder even if it were larger than the divisor, and the dividend was obtained in the conventional manner. The worst of it was that the pupil felt sure that his example was right, for it seemingly proved. Here, of course, the remedy is to put more emphasis on that step of the long division process in which the pupil should compare his remainder with his divisor.

But it is the purpose of this study to show that the errors made in long division can be identified. Corrective procedure is another task which a study such as this one helps to direct into the proper channel.

To get back again to the discussion of the attainment of 100% accuracy. This study emphasizes the fact that the task of achieving 100% accuracy is a tremendous one; but by being on the job all the time and going at the task in a scientific manner, the idea can, at least, be approached.

ILLUSTRATIVE ERRORS

To show that a teaching plan is needed, the writer is adding examples illustrative of particular errors. The set given is not exhaustive but is simply typical of what children will do if not carefully taught. The examples are grouped according to the grade in which they were done, more to show that pupils in all the grades need watching, rather than that the wrong examples are typical of the grade in which they are listed.

Grade VIII

Illustrative Errors

```
         51
a.  17)109
        85
        24
        17
         7
```

The pupil started with the quotient figure too small and then, having a remainder greater than the divisor, put another figure in the quotient. Proof by multiplying would have shown this pupil how wrong he was. (Correct answer, 6 and 7r).

```
        3116
b.  18)7502
       54
       21
       18
        30
        18
        122
        108
         14
```

The mistake made in example b. is similar to that made in example a., but in this case the extra digit is inserted between figures in the quotient. (Correct answer, 416 and 14r).

```
         22
c.  19)385  (0)
       38
        50
        38
        12
```

In example c., the pupil added a zero to the dividend, seemingly in order to obtain a number large enough to contain the divisor a second time. (Correct answer, 20 and 5r).

Grade VIII

Illustrative Errors (Continued)

```
        79
d.  63)5261
       441
        851
        567
        284
```

The pupil, starting with a quotient figure too small, disregards the fact that the remainder is greater than the divisor and that the second remainder is several times as large as the divisor. (Correct answer, 83 and 32r).

Grade VII

Illustrative Errors

```
         9
e.  21)248
       189
        59
```

Divisor contained in first two figures of dividend but first three used. A remainder over twice as big as the divisor disregarded. (Correct answer, 11 and 17r).

```
        25
f.  19)385
       38
```

In this example the pupil took the 5 which should have been the remainder and placed it as the next quotient figure. (Correct answer, 20 and 5r).

```
       417
g. 18)7502
      72
       30
       18
       122
       126
         6
```

In example g. the third quotient figure was too large, making the third partial product larger than the remainder, yet the pupil "subtracted". (Correct answer, 416 and 14r).

```
          85
h.  63)5261
       504
        221
        189
         321
         315
           6
```

The pupil did not set down the 3 which was the second quotient figure, but he multiplied the divisor by the 3. Then, after subtracting, probably noticing that he should have two figures in the quotient, he brought down the 1 a second time and obtained 5 as the quotient figure. (Correct answer, 83 and 32r).

Grade VII

Illustrative Errors (Continued)

```
        30
i.  16)583
        48
        10~~3~~
```

Zero put as last quotient figure, after crossing off one figure (3) of the remainder. (Correct answer, 36 and 7r).

```
         10
j.  17)109
        1 7
          2
```

The pupil ignored the zero between the 1 and the 9 in 109. Just why the zero was attached to the quotient is hard to tell. (Correct answer, 6 and 7r).

Grade VI

Illustrative Errors

```
        56
k.  59)327
       295
        32
       354
       278
```

The first quotient figure was put over the second instead of the third dividend figure. Then the pupil took 6 as the next quotient figure, probably because the 5 of 59 is contained 6 times in 32. He found no difficulty in subtracting 354 from 32 and was not bothered by a remainder over four times the divisor. (Correct answer, 5 and 32r).

```
         4168
l.  18)7502
       72
        30
        18
        122
        108
         14
         143
           3
```

This pupil had the example correct if he stopped when he had 416 and 14r; but he put another 8 on the quotient, multiplied 18 x 8 incorrectly, and then subtracted 143 from 14 to get for his remainder, 3. (Correct answer, 416 and 14r).

Grade VI

Illustrative Errors (Continued)

```
       103
m.  31)3224
       31
       124
        93
        31
```

Here a remainder of 31, equal to the divisor, did not suggest to the pupil that the quotient figure should be increased by 1. (Correct answer, 104 and 0r).

Grade V

Illustrative Errors

```
       10    Proof
n.  13)165   13
      130     10
       35    130
              35
             165

       319
o.  18)7502
       54     319
      210      18
       18    2552
     1922     319
      162    5742
     1760   +1760
             7502
```

In examples n. and o. the pupils proved their work by multiplying. They were evidently satisfied that their answers were right, since their proof produced the dividend. This particular error was quite common in grade V, showing that more emphasis on comparing the remainder with the divisor should be given.

(Correct answers, n. 12 and 9r
o. 416 and 14r)

```
       12      Proof
p.  17)109      12
       17       17
       39       84
       34       12
        5   x  104
                 5
               109
```

This is an illustration of an error in the proof, making the answer seem to be correct. The error in the proof is in not carrying 1 when 84 were added.
12

(Correct answer, 6 and 7r).

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